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Chaotic quantization and the parameters of the standard model

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Abstract. In the chaotic quantization approach one replaces the Gaussian white noise of the Parisi-Wu approach of stochastic quantization by a deterministic chaotic process on a very small scale. We consider suitable coupled chaotic noise processes as generated by Tchebyscheff maps, and show that the vacuum energy of these models is minimized for coupling constants that coincide with running standard model couplings at energy scales given by the known fermion and boson masses. Chaotic quantization thus allows to predict fundamental constants of nature from first principles. At the same time, it provides a natural framework to understand the dynamical origin of vacuum energy in our universe.

1. Introduction

The important role of chaos in quantum field theories and string theories has been emphasized in various recent papers and books [1]–[8]. ‘t Hooft conjectures that the ultimate theory underlying quantum mechanical behaviour is a dissipative one exhibiting complex behaviour [1]. Damour, Henneaux, Julia and Nicolai [2] emphasize that M-theory, the hypothetical theory of all interactions, is intrinsically chaotic. Kogan and Polyakov have recently studied chaotic renormalization flow and Feigenbaum universality in string theory [3]. An important way how chaos can enter into quantum field theories is via the so-called ’chaotic quantization’ method [4]. Here one assumes that the noise used for stochastic quantization has a dynamical (deterministic chaotic) origin. It has been recently pointed out [5, 6] that this method allows for some very precise predictions of standard model parameters. Also, Biró, Müller and Matinyan have recently shown that chaotic classical Yang Mills theories [7] can ’quantize themselves’, i.e. the noise used for stochastic quantization can be intrinsically generated by the strongly chaotic behaviour of the classical field equation [8].

In the following sections, we will first review how to extend the stochastic quantization approach of Parisi and Wu [9, 10] to a chaotic quantization method [4]. Then we summarize how this method can yield predictions on the fundamental constants of nature, such as coupling constants, masses, and mixing angles of the standard model, using a simple principle, the minimization of vacuum energy [5, 6]. Indeed, one of the main features of the chaotic quantization approach is that it naturally produces a non-vanishing expectation of vacuum energy, due to the potentials of the underlying deterministic chaotic theory on the smallest scales. This vacuum energy may well stand in relation to the dark energy that is currently observed in our universe [11]. In this paper we just review the main ideas and results, much more details can be found in [6].

2. Chaotic quantization

Let us first recall the stochastic quantization method, then we generalize it to chaotic quantization. A field theory is usually determined by some action functional $S[\phi]$. The field ϕ is a function of the space-time coordinates and may, in general, have many components. The classical field equation can be written as

$$\frac{\delta S}{\delta \phi} = 0, \tag{1}$$

meaning that the action has an extremum.

In the Parisi-Wu appraoch of stochastic quantization one proceeds from the classical field equation to a quantized theory by means of the following Langevin equation:

$$\frac{\partial}{\partial t}\phi(x,t) = -\frac{\delta S}{\delta \phi}(x,t) + L(x,t) \tag{2}$$

Here $x = (x^1, x^2, x^3, x^4) = x^\mu$ is a point in Euclidean space-time, t denotes a fictitious time variable (different from the physical time x^4), and $L(x,t)$ denotes spatio-temporal Gaussian white noise, δ-correlated in both space-time x and fictitious time t.

The fictitious time t is just introduced as an artificial fifth coordinate. It is different from the physical time. What is of physical relevance is the stationary solution of the Langevin equation in the limit $t \to \infty$. It is the quantized field, a stochastic process. All quantum mechanical expectations of the field $\phi(x)$ can be calculated as expectations with respect to the realizations of the Langevin process in the limit $t \to \infty$.

The action S of the entire standard model can be 2nd-quantized in this way (at least in principle). For each standard model field, there is a corresponding noise field. One may then ask: Where do these rapidly fluctuating noise fields ultimately come from? Could they have dynamical origin? The idea of chaotic quantization is that the noise fields used for second quantization are not truely random but generated by a rapidly fluctuating deterministic chaotic process. One can, for example, generate the noise variables at each space-time point by a chaotic map T. If this map has the so-called φ-mixing property [12], then it can be rigorously proved that rescaled sums of iterates generate the Wiener process (= Brownian motion) on large scales, regarding the initial value as a random variable. In other words, the fast chaotic dynamics looks locally like Gaussian white noise if seen from a larger scale. Hence, on large scales ordinary quantum field theoretical behaviour is generated if chaotic 'noise' is used for quantization. Only on small scales (the Planck scale or below) there is much more complex behaviour and nontrivial correlations.

A simple model is to generate the noise by Tchebyscheff maps. One can actually show that Tchebyscheff maps of order N

$$\Phi_{n+1} = T_N(\Phi_n), \quad \Phi_0 \in [-1,1], \tag{3}$$

are φ-mixing [12]. In nonlinear dynamics, the T_N are standard examples of chaotic maps, just similar as the harmonic oscillator is a standard example in linear dynamics. One has $T_2(\Phi) = 2\Phi^2 - 1$ and $T_3(\Phi) = 4\Phi^3 - 3\Phi$, generally $T_N(\Phi) = \cos(N \arccos \Phi)$. There is sensitive dependence on initial conditions for $N \geq 2$: Small perturbations in the initial values will lead to completely different trajectories in the long-term run. The maps are conjugated to a Bernoulli shift with an alphabet of N symbols. This means, in suitable coordinates the iteration process is just like shifting symbols in a symbol sequence.

Most important for our purposes is the following property: One can show that the Tchebyscheff maps have least higher-order correlations among all smooth systems conjugated to a Bernoulli shift, and are in that sense closest to Gaussian white noise, as close as possible for a smooth deterministic system [13, 14]. Any other map has more higher-order correlations. What does this mean for chaotic quantization? It is plausible that if nature chooses to generate Gaussian white noise by something deterministic chaotic on the smallest quantization scales, it aims for making the small-scale deviations from ordinary quantum mechanics as small as possible. This automatically leads to Tchebyscheff maps. A graph theoretical method for this type of 'deterministic noise' has been developed in [13,14].

3. Coupled chaotic noise fields

Once we assume that the noise fields used for quantization are dynamical in origin, it is natural to allow for some coupling between neighbored noise fields. In string theory, in a perturbative approach, point particles are replaced by little extended 1-dimensional objects, strings. Now if we go to strings in the standard model space, it's natural to also proceed to 'chaotic strings' in the corresponding chaotic noise space used for second quantization. This is illustrated in Fig. 1. Each ordinary string might be 'shadowed' by a corresponding chaotic noise string used for second quantization purposes.

Among the many models that can be chosen to generate a coupled chaotic dynamics on a small scale certain criteria should be applied to select a particular system. First of all, for vanishing spatial coupling of the chaotic 'noise' one wants to have strongest possible random behavior with least possible higher-order correlations, in order to be closest to the Gaussian limit case (which corresponds to ordinary path integrals on a large scale). This selects as a local dynamics Tchebyscheff maps $T_N(x)$ of N-th order ($N \geq 2$). Now let us discuss possible ways of spatially coupling the chaotic noise. Although in principle all types of coupling forms can be considered, physically it is most reasonable that the coupling should result from a Laplacian coupling rather than some other coupling, since this is the most relevant coupling form in quantum field and string theories. This leads to coupled map lattices of the nearest-neighbour coupling form. The resulting coupled map lattices can then be studied on lattices of arbitrary dimension, but motivated by the

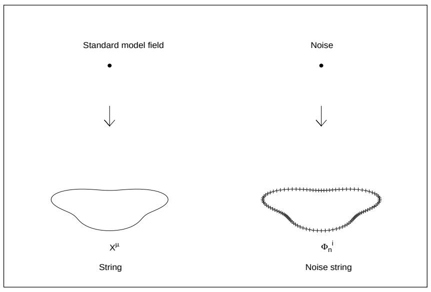

Fig. 1. In string theory, point particles are replaced by strings. For symmetry reasons, we may then also replace the chaotic point noise fields used for 2nd quantization by little extended noise objects, 'chaotic strings'.

fact that ordinary strings are 1-dimensional objects we will here consider 1-dimensional structures, although higher-dimensional chaotic objects ('chaotic branes') can be studied as well [15, 6]. We end up with coupled Tchebyscheff maps of the form

$$\Phi_{n+1}^i = (1-a)T_N(\Phi_n^i) + s\frac{a}{2}(T_N^b(\Phi_n^{i-1}) + T_N^b(\Phi_n^{i+1})), \tag{4}$$

where i is a 1-dimensional lattice coordinate, $a \in [0,1]$ is a coupling constant, $s = \pm 1$, and b takes on values 0 or 1 ($T^0(\Phi) = \Phi, T^1(\Phi) = T(\Phi)$). The chaotic string dynamics (4) is deterministic chaotic, spatially extended, and strongly nonlinear. The field variable Φ_n^i is physically interpreted in terms of rapidly fluctuating virtual momenta in units of some arbitrary maximum momentum scale.

It is easy to see that for odd N the statistical properties of the coupled map lattice are independent of the choice of s (since odd Tchebyscheff maps satisfy $T_N(-\Phi) = -T_N(\Phi)$), whereas for even N the sign of s is relevant and a different dynamics arises if s is replaced by $-s$. Hence, restricting ourselves to the 'ground states' of our chaotic string oscillators, i.e. $N = 2$ (even maps) and $N = 3$ (odd maps), in total 6 different chaotic string theories arise, characterized by $(N,b,s) = (2,1,+1),(2,0,+1),(2,1,-1),(2,0,-1)$ and $(N,b) = (3,1),(3,0)$. For easier notation, we have labeled these chaotic string theories as $2A, 2B, 2A^-, 2B^-, 3A, 3B$, respectively. Chaotic strings can also be regarded as discrete versions of self-interacting scalar fields that are homogeneous in all but one space-time direction.

4. Vacuum energy density due to chaotic quantization effects

Though the chaotic string dynamics is dissipative, one can formally introduce potentials that generate the discrete time evolution. For example, the 3rd-order Tchebyscheff dynamics can be written as

$$\Phi_{n+1} - \Phi_n = 4\Phi_n^3 - 4\Phi_n. \tag{5}$$

This equation formally describes a discrete momentum change (= force) generated by the self-interacting potential

$$V^{(3)}(\Phi) = \left(-\Phi^4 + \frac{3}{2}\Phi^2\right) + \frac{1}{2}\Phi^2 + C, \tag{6}$$

the force being given by $-\frac{\partial V^{(3)}}{\partial \Phi}$. One can now incorporate symmetry considerations between $+T_N$ and $-T_N$ [6], with the result that there are two interesting observables to look at for chaotic strings, the expectation of the self energy given by

$$V^{(2)}(a) = -\frac{2}{3}\langle\Phi^3\rangle + \langle\Phi\rangle \quad (N=2) \tag{7}$$

$$V^{(3)}(a) = -\langle\Phi^4\rangle + \frac{3}{2}\langle\Phi^2\rangle \quad (N=3), \tag{8}$$

and the expectation of the interaction energy given by

$$W(a) = \frac{1}{2}\langle\Phi^i\Phi^{i+1}\rangle. \tag{9}$$

All expectations can be calculated as long-term averages over n and i for random initial conditions.

Note that in quite a natural way there is vacuum energy associated with our deterministic chaotic dynamics, given by the above equations. Could this vacuum energy have something to do with the dark energy that makes up most of the energy density (70%) of our universe, as recently confirmed by various astronomical observations? It might indeed. The absolute unit of the vacuum energy of our chaotic noise fields is not fixed in our theory. But most naturally, if we quantize a particle of mass m then one would expect that the corresponding vacuum energy of the corresponding noise field yields a similar energy contribution, since the potential V simply generates chaotic fluctuations of the particle momentum mc ([6], chapter 5). Hence it is most natural to conjecture that the vacuum energy density generated by chaotic quantization effects has the same order of magnitude as the mass density of particles in our universe, since for each particle there is a corresponding noise field used for quantization. This could point towards a possible solution of the 'cosmological coincidence' problem [16].

The expectations of vacuum energy (7)–(9) depend on the coupling constant a in a nontrivial way and are like a 'thermodynamic potential' of vacuum fluctuations. $V(a)$ and $W(a)$ (and their sum [6]) can be easily numerically determined by iterating the coupled map and averaging over all i and n.

A helpful physical interpretation of the coupled map dynamics is as follows. Suppose we regard Φ_n^i to be a fluctuating virtual momentum component that can be associated with a hypothetical 'noise' particle i at time n, all particles i being ultimately responsible for a dynamical state underlying dark energy. n can be either interpreted as fictitious time or as physical time, both interpretations make sense [6]. Neighbored particles i and $i-1$ can exchange momenta due to the Laplacian coupling of the coupled map lattice. We

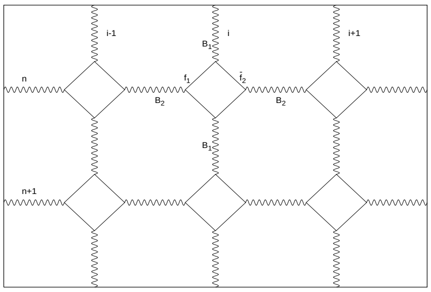

Fig. 2. Interpretation of the coupled map dynamics in terms of fluctuating momenta exchanged by fermions $f_1, \bar{f}_2$ and bosons B_1, B_2. Φ_n^i corresponds to the momentum in the fermion loop.

may actually associate a fermion-antifermion pair $f_1, \bar{f}_2$ with each cell i. In units of some arbitrary energy scale p_{max}, the particle has momentum Φ_n^i, the antiparticle momentum $-\Phi_n^i$. They interact with particles in neighbored cells by exchange of a (hypothetical) gauge boson B_2, then they annihilate into another boson B_1 until the next vacuum fluctuation takes place. This can be (symbolically!) described by the Feynman graph in Fig. 2. We call this graph a 'Feynman web', since it describes an extended spatio-temporal interaction state of space-time, to which we have given a standard model-like interpretation. Note that in this interpretation a is a (hypothetical) standard model coupling constant, since it describes the strength of momentum exchange of neighbored particles. At the same time, a can also be regarded as an inverse metric in the 1-dimensional string space, since it determines the strength of the Laplacian coupling.

What is now observed numerically for the various chaotic strings is that the interaction energy $W(a)$ has zeros and the self energy $V(a)$ has local minima for string couplings a that numerically coincide with running standard model couplings $\alpha(E)$, the energy being given by

$$E = \frac{1}{2} N \cdot (m_{B_1} + m_{f_1} + m_{f_2}). \tag{10}$$

Here N is the index of the chaotic string theory considered, and m_{B_1}, m_{f_1}, m_{f_2} denote the masses of the standard model particles involved in the Feynman web interpretation. The surprising observation is that rather than yielding just some unknown exotic physics, the chaotic string spectrum appears to reproduce the masses and coupling constants of the known quarks, leptons and gauge bosons of the standard model with very high precision. Gravitational and Yukawa couplings are observed as well. The chaotic dynamics can be used to fix the fundamental constants of nature by a simple principle, the minimization of vacuum energy.

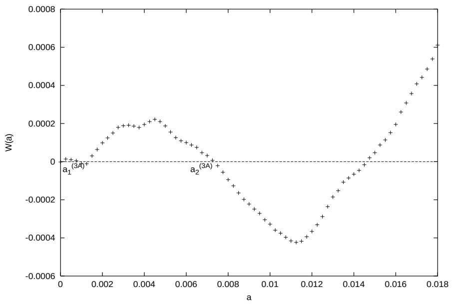

Fig. 3. Numerically determined interaction energy of the 3A string.

5. Some numerical results

Let us now present some examples of numerical results (much more numerical evidence on the validity of eq. (10) can be found in [6]). Fig. 3 shows the interaction energy $W(a) = \frac{1}{2}\langle \Phi_n^i \Phi_n^{i+1} \rangle$ of the chaotic $3A$ string in the low-coupling region. We numerically find two zeros of $W(a)$ with $W'(a) < 0$ in the low-coupling region:

$$\begin{aligned} a_1^{(3A)} &= 0.0008164(8) \\ a_2^{(3A)} &= 0.0073038(17) \end{aligned}$$

Remarkably, the zero $a_2^{(3A)}$ appears to coincide with the running fine structure constant $\alpha_{el} \approx 1/137$, evaluated at an energy scale given by 3 times the electron mass. We find the amazing numerical coincidence

$$a_2^{(3A)} = \alpha_{el}(3m_e), \tag{11}$$

the energy scale $3m_e$ being in agreement with eq. (10) with $f_1 = e^-, \bar{f}_2 = e^+$ (electrons and positrons) and B_1 massless. Eq. (11) is satisfied with 4 digits precision (more details in [6]).

For the other zero, $a_1^{(3A)}$, one finds that it coincides, with similar precision, with the electric coupling constant

$$a_1^{(3A)} = \alpha_{el}^d(3m_d) = \frac{1}{9}\alpha_{el}(3m_d), \tag{12}$$

of d-quarks.

But what about u-quarks and neutrinos? The interaction energy of the 3B string is plotted in Fig. 4. Again there are two zeros with negative slope in the low-coupling region,

$$\begin{aligned} a_1^{(3B)} &= 0.0018012(4) \\ a_2^{(3B)} &= 0.017550(1) \end{aligned}$$

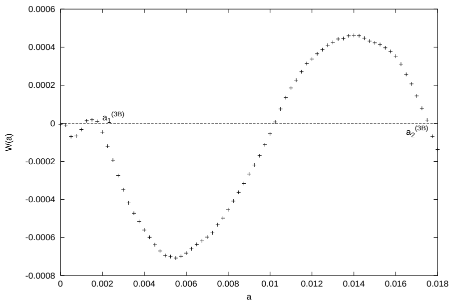

Fig. 4. Same as Fig. 3, but for the 3B string.

These, again with a precision of 4 digits, are found to coincide with weak interaction strengths of u-quarks and electron-neutrinos, if these are assumed to be there in addition to electrically interacting d-quarks and electrons (see [5, 6] for the details). At latest at this stage one notices that all this can't be a random coincidence. One can have one random coincidence, say of the fine structure constant with the zero $a_2^{(3A)}$, but not 3 other random coincidences at the same time! We are thus lead to the conclusion that the smallest zeros of the interaction energy of the 3A and 3B string fix the electroweak coupling strengths at the smallest fermionic mass scales.

Similarly, one numerically finds that the smallest zeros of the interaction energies of the $N = 2$ strings coincide with strong couplings at the smallest bosonic mass scales. In particular, the $W^\pm$ mass comes out correctly, and a Higgs mass prediction of (154.4 ± 0.5) GeV is obtained (see [5, 6] for more details).

Another interesting observable is the self-energy $V^{(N)}(a)$ of the strings. Typically the self-energies $V^{(N)}(a)$ have lots of local minima. As an example, Fig. 5 shows $V^{(2)}(a)$ for the 2A/B string. For all strings, one numerically observes that log-oscillatory behaviour with period N^2 sets in for small a, hence e.g. for the $N = 2$ strings all minima are only determined up to an arbitrary power of 4. In other words, they are only determined modulo 4. Remarkably, one observes minima that coincide with Yukawa and gravitational couplings of the known fermions modulo 4 (Fig. 5). The minima b_2, b_6, b_{10} turn out to coincide with Yukawa couplings modulo 4 of the heavy fermions τ, b, c

$$b_i = \alpha_{Yu} = \frac{1}{4}\alpha_2(m_H + 2m_f)\left(\frac{m_f}{m_W}\right)^2 \cdot 4^n, \tag{13}$$

where $f = \tau, b, c$, respectively,[1] and for the light fermions one observes that the self energy has local minima for couplings that coincide with gravitational couplings modulo 4. We

[1] A t-quark minimum is also observed, but outside the low-coupling region.

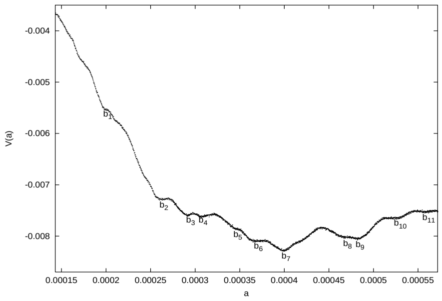

Fig. 5. Local minima of the self-energy of the 2A/B string fixing the fermion masses.

numerically observe for $i = 1,4,7,8,9$

$$b_i = \alpha_G = \frac{1}{2}\left(\frac{m_f}{m_{Pl}}\right)^2 \cdot 4^n, \tag{14}$$

where $f = \mu, e, d, u, s$, respectively. Solving for m_f, one can thus get fermion mass predictions modulo 2. The relevant power of 2 can then be obtained from other minima and additional symmetry considerations [6]. The remaining minima in Fig. 5 yield neutrino mass predictions [5,6].

6. Fixing standard model parameters

What is the theory behind all these numerically observed coincidences? The principal idea is very simple. At a very early stage of the universe, where standard model parameters are not yet fixed and ordinary space-time may not yet exist as well, pre-standard model couplings are realized as coupling constants a in the chaotic noise space. The parameters are then fixed by an evolution equation (a renormalization flow) of the form

$$\dot{a} = const \cdot W(a) \tag{15}$$

(see Fig. 6), respectively

$$\dot{a} = -const \cdot \frac{\partial V}{\partial a}, \tag{16}$$

where we assume that the constant *const* is positive. The equations make *a priori* arbitrary standard model couplings a evolve to the stable zeros of $W(a)$, respectively to the local minima of $V(a)$. There they will stay forever, since any other value of the fundamental constants is energetically less favourable.

Our main conclusion could be formulated as follows. The standard model appears to have evolved to a state where its free parameters minimize the vacuum energy associated with the chaotic noise fields. If this chaotic dynamics keeps on evolving today, then the

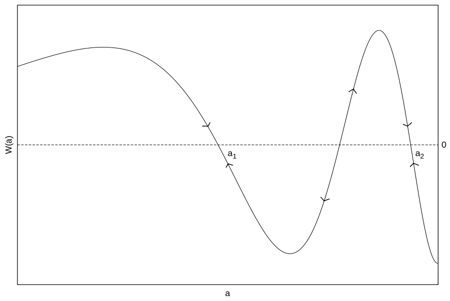

Fig. 6. Approach of arbitrary initial couplings to stable zeros a_1, a_2 of the interaction energy (schematic plot).

fundamental constants are in fact stabilized by the local minima of the energy landscape associated with the chaotic dynamics. Any fluctuation to other values drives the fundamental 'constants' immediately back to the equilibrium state, according to eqs. (15) and (16). The total expectation of vacuum energy obtained from the chaotic dynamics in this way may well correspond to the dark energy seen in the universe today.

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