NL2599 String theory

String theories (Green et al., 1987, Davies & Brown, 1988; Polchinski, 1998; Gauntlett, 1998; Kaku, 1999) were developed in the hope to find a unified description of all forces in nature. The aim is to combine electroweak, strong, and gravitational interactions into one consistent theory. The basic idea is that particles are not point-like but little 1-dimensional extended objects, strings. These vibrate (similar to a violin string) in various modes and can move in space-time. The various particle states that we observe in nature should then all be explained as suitable excitations of suitable strings. Interaction processes between particles are described by splitting and merging of strings (Fig. 1). Due to the finite size of strings, many of the divergences that occur for point-like particles can be avoided. The size of the string is very small, typically of the order of a few Planck lengths, so on large scales string-like particles look like point particles.

Different types of boundary conditions are possible, there are open strings and closed strings. The simplest example is a bosonic string. In suitable coordinates the equation of motion is simply a wave equation of the form

$$\frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\tau^2} = C^2 \frac{\partial^2 X^\mu(\sigma,\tau)}{\partial\sigma^2}. \tag{1}$$

Here X^μ is the coordinate of the string-like particle in d-dimensional space-time ($\mu = 0, 1, \ldots, d-1$), σ and τ denote an internal position and time variable parametrizing the string, and C is a constant.

The general solution can be written as a mode expansion of the form

$$X^\mu(\sigma,\tau) = x^\mu + \dot{x^\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^j e^{-i\frac{n\pi C\tau}{L}}\cos\frac{n\pi\sigma}{L}. \tag{2}$$

The string parameter $\sqrt{\alpha'}$ describes the approximate scale where string effects become relevant, and L is the length of the string. Unlike a violin string, this string is not tied down at the end points but it can move freely through spacetime while it oscillates.

So far this is a classical string. As usual, we have to 1-st quantize (and finally also 2-nd quantize) the string dynamics. First quantization is done by letting the coefficients α_n^j of the above normal mode expansion satisfy suitable commutator relations. Second quantization of strings, up to now, is still an active area of research with many open questions (Kaku, 1999).

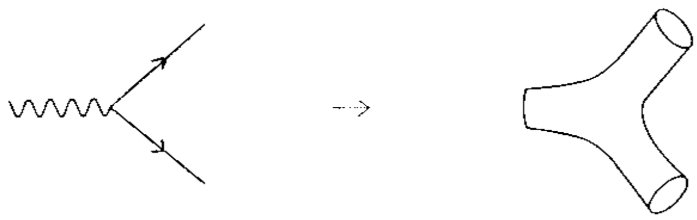

Figure 1: Point-like interactions in Feynman graphs are smoothed out in string theory.

The quantization of bosonic strings yields unpleasant surprises, so-called ghosts. These are quantum states with negative norm. One wants to avoid these unphysical states. This turns out to be only possible if the dimension of space-time is $d = 26$. Consequently, one has to assume that the 22 extra dimenions are "compactified", i.e. curled up on small circles that are so small that we don't notice them in daily life.

But bosonic string theories have another problem. The lowest excitation mode (the ground state) is a so-called "tachyon", a state for which the mass squared is negative. To avoid this unphysical (unstable) ground state, one proceeds to supersymmetric string theories, in short "superstring theories". In these theories, for each boson there is a corresponding fermionic partner and vice versa. Technically, one introduces anti-commuting coordinates in addition to commuting ones. For supersymmetric theories ghost states occur as well, but can now be avoided if the number of space-time dimensions is $d = 10$. So to describe low-energy physics, again 6 extra dimensions have to be compactified. One has found 5 different superstring theories, which are denoted as type I, IIA, IIB, HET $E_8 \times E_8$, HET $SO(32)$. They differ in the degree of supersymmetry, the underlying gauge symmetry (if any), whether the strings are closed or open etc.

Nowadays the 5 superstring theories are regarded as special limit cases of a more fundamental theory, called M-theory. This contains the 5 superstring theories as well as an 11-dimensional theory as special cases. These special cases are marked as the edge points of the diagram in Fig. 2. Unfortunately,

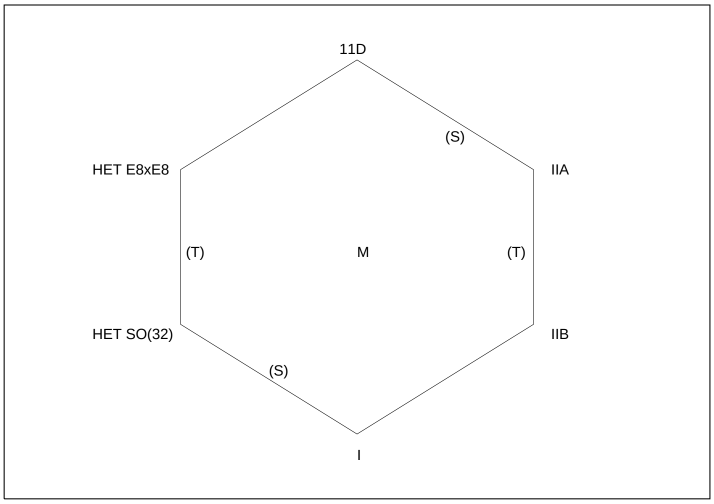

Figure 2: A schematic diagram of M-theory with its known string theory limits and the 11-dimensional theory. Some of the dualities are shown as well.

it is not known how M-theory should be formulated in full generality and how it should be 2nd quantized. It is not even clear what the fundamental objects are that this theory describes. So a lot of work needs still to be done.

Some of the string theories contained in M-theory are connected by so-called duality transformations. S-duality essentially means that a string theory with a small coupling constant α describes the same physics as another string theory with a large coupling given by $1/\alpha$. T-duality means invariance of the physics if a compactified dimension of radius R is replaced by another one of radius $1/R$.

In recent years, much emphasis of reasearch has concentrated on higher-dimensional objects contained in string and M-theory, so-called D-branes. Open strings can end and start on D-branes. Remember that in electromagnetism there are point charges, which are the sources of the electromagnetic field. D-branes are in a sense higher-dimensional generalizations of these point charges.

As already mentioned, to proceed to our 4-dimensional world 6 of the dimensions of superstring theory (or 7 of those of M-theory) have to be compactified. The way how this should be done is somewhat unclear. One needs to compactify on special types of compact manifolds, so-called Calabi

Yau manifolds, to preserve what is generally believed to be the right amount of supersymmetry. The compactification step is clearly necessary for string theory to make contact with the real world, but almost all relevant low energy predictions depend on the numerous possibilities how to compactify. This severely undermines the predictive power of string theories.

In principle, the ultimate theory should predict everything we want to know about particle physics, such as the values of the coupling constants of the four different forces, or the masses of all fermions and bosons. In string theories, the numerical values of gauge couplings (e.g. the fine structure constant α_{el}, which is about 1/137 at low energies) can be related to vacuum expectations of a scalar field contained in string theories, the dilaton field. But this vacuum expectation depends in an unknown way on second quantization effects and has not been predicted so far. As G. 't Hooft puts it in his book ('t Hooft, 1997), string theory, at least in its present stage, has similarities with a very uncomplete piece of furniture: "Imagine that I give you a chair while explaining that the legs are still missing, and that the seat, back, and armrest will perhaps be delivered soon; whatever I did give you, can I still call it a chair?"

While typical equations for (free) strings, such as eq. (1), are linear, a recent proposal is to amend ordinary strings (evolving in a regular way) by nonlinear versions of strings, so-called chaotic strings (Beck, 2002). These evolve in a deterministic chaotic way. In this approach each ordinary string is shadowed by a chaotic string, which yields the "noise" for second quantization via the so-called stochastic quantization method. Mathematically, chaotic strings consist of 1-dimensionally coupled Tchebyscheff maps, a very nonlinear and strongly self-interacting theory, which describes a kind of "turbulent quantum state" on a small (quantum gravity) scale. It turns out that the vacuum energy of chaotic strings is minimized for observed standard model parameters, i.e. in this extended approach to second quantization concrete predictions for vacuum expectations of dilaton-like fields and hence on masses and coupling constants can be given.

Christian Beck

see also quantum field theory, elementary particles, general relativity

Further reading:

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