Fluctuation relations for anomalous dynamics

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DPG Spring Meeting, 18 March 2011

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Motivation: Fluctuation relations

Consider a (classical) particle system evolving from some initial state into a nonequilibrium steady state.

Measure the probability distribution $\rho(W_t)$ of entropy production W_t during time t:

$$\ln \frac{\rho(W_t)}{\rho(-W_t)} = W_t$$

transient fluctuation relation (TFR)

Evans, Cohen, Morriss (1993); Gallavotti, Cohen (1995)
(basic idea for dynamical systems)

why important? Of *very general validity* and

1. generalizes the Second Law to small noneq. systems
2. yields nonlinear response relations
3. connection with fluctuation dissipation relations (FDR)

Fluctuation relation for Langevin dynamics

example: Check TFR for the overdamped Langevin equation

$$\dot{x} = F + \xi(t) \qquad \text{(set all irrelevant constants to 1)}$$

with constant field F and Gaussian white noise $\xi(t)$.

Entropy production is equal to (mechanical) work, $W_t = Fx(t)$, hence $\rho(W_t) = F^{-1}\rho(x,t)$; remains to solve corresponding Fokker-Planck eq. for initial condition $x(0) = 0$:

the position pdf is Gaussian,

$$\rho(x,t) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{(x-<x>)^2}{2\sigma_x^2}\right)$$

easy to see:

TFR holds if $< W_t >=< \sigma_{W_t}^2 > /2$

i.e., $\exists$ fluctuation-dissipation relation 1 **(FDR1)** $\Rightarrow$ **TFR**

see, e.g., van Zon, Cohen, PRE (2003)

TFR for correlated internal Gaussian noise

consider overdamped generalized Langevin equation

$$\int_0^t dt'\dot{x}(t')K(t-t') = F + \xi(t)$$

with internal Gaussian noise defined by the **FDR2**

$$< \xi(t)\xi(t') > \sim K(t-t') \, ,$$

which is correlated by $K(t) \sim t^{-\beta}$, $0 < \beta < 1$

$\rho(W_t) \sim \rho(x,t)$ is Gaussian; solving for $x(t)$ in Laplace space yields **subdiffusion**

$$< \sigma_x^2 > \sim t^{\beta}$$

by preserving **FDR1**,

$$< W_t > = < \sigma_{W_t}^2 > /2$$

for correlated internal Gaussian noise ∃ TFR

TFR for correlated external Gaussian noise

consider overdamped generalized Langevin equation

$$\dot{x} = F + \xi(t)$$

with correlated Gaussian noise defined by

$$< \xi(t)\xi(t') >\sim |t - t'|^{-\beta} \, , \; 0 < \beta < 1 \, ,$$

which is external, because there is **no FDR2**

$\rho(W_t) \sim \rho(x, t)$ is again Gaussian but here with **superdiffusion** by **breaking FDR1**:

$$< W_t >\sim t \quad , \quad < \sigma^2_{W_t} >\sim t^{2-\beta}$$

yields the **anomalous TFR**

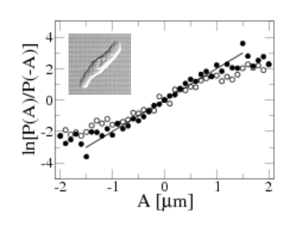
$$\ln \frac{\rho(W_t)}{\rho(-W_t)} = \mathbf{C}_\beta \mathbf{t}^{\beta-\mathbf{1}} W_t \quad (0 < \beta < 1)$$

note: pre-factor on rhs *not equal to one* and *time dependent*

Relations to experiments

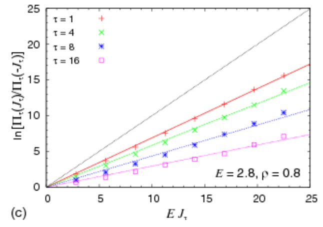
$$\ln \frac{\rho(W_t)}{\rho(-W_t)} = \frac{\mathbf{C}_\beta}{\mathbf{t}^{1-\beta}} W_t \quad (0 < \beta < 1)$$

experiments on slime mold:



Hayashi, Takagi, J.Phys.Soc.Jap. (2007)

computer simulation on glassy lattice gas:



Sellitto, PRE (2009)

⇒ anomalous fluctuation relation important for glassy dynamics

TFR for other anomalous stochastic processes

- consider the Langevin equation

$$\dot{x} = F + \xi(t)$$

with white Lévy noise $\rho(\xi) \sim \xi^{-1-\alpha}\ (\xi \to \infty)$, $0 \leq \alpha < 2$,
breaking FDR1; solving a space-fractional Fokker-Planck eq.
we recover the result of Touchette, Cohen, PRE (2007)

$$\lim_{w \to \pm\infty} g_t(w) = \lim_{w \to \pm\infty} \frac{\rho(W_t = wF^2 t)}{\rho(W_t = -wF^2 t)} = 1$$

i.e., large fluctuations are *equally possible*

- consider the subordinated Langevin equation

$$\frac{dx(u)}{du} = F + \xi(u) \quad , \quad \frac{dt(u)}{du} = \tau(u)$$

with Gaussian white noise $\xi(u)$ and white Lévy stable noise
$\tau(u) > 0$, which **preserves** a generalized **FDR2**
by solving the corresponding time-fractional Fokker-Planck eq.
we recover the conventional TFR

Summary

- TFR tested for three fundamental types of **anomalous stochastic dynamics**:
  1. Gaussian stochastic processes with correlated noise: TFR holds for internal noise, mild violation for external one
  2. strong violation of TFR for space-fractional (Lévy) dynamics
  3. TFR holds for time-fractional dynamics

$$\textbf{FDR2} \Rightarrow \textbf{FDR1} \Rightarrow \textbf{TFR}$$

- same results obtained for a particle confined in a harmonic potential dragged by a constant velocity

Reference:

A.V. Chechkin, R. Klages, Fluctuation relations for anomalous dynamics, J. Stat. Mech. L03002 (2009)