Normal and Anomalous Fluctuation Relations for Gaussian Stochastic Dynamics

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Motivation: Fluctuation relations

Consider a (classical) particle system evolving from some initial state into a nonequilibrium steady state.

Measure the probability distribution $\rho(\xi_t)$ of entropy production ξ_t during time t:

$$\ln \frac{\rho(\xi_t)}{\rho(-\xi_t)} = \xi_t$$

Transient Fluctuation Relation (TFR)

Evans, Cohen, Morriss (1993); Gallavotti, Cohen (1995)

why important? of *very general validity* and

1. generalizes the Second Law to (small) systems in nonequ.
2. connection with fluctuation dissipation relations
3. can be checked in experiments (Wang et al., 2002)

Fluctuation relation for Langevin dynamics

warmup: check TFR for the overdamped Langevin equation

$$\dot{x} = F + \zeta(t) \qquad \text{(set all irrelevant constants to 1)}$$

for a particle at position x with constant field F and noise ζ.

entropy production ξ_t is equal to (mechanical) work $W_t = Fx(t)$ with $\rho(W_t) = F^{-1}\varrho(x,t)$; remains to solve corresponding Fokker-Planck equation for initial condition $x(0) = 0$:

the position pdf is Gaussian,

$$\varrho(x,t) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{(x-<x>)^2}{2\sigma_x^2}\right)$$

straightforward:

(work) TFR holds if $<x>= \sigma_x^2/2$

and $\exists$ fluctuation-dissipation relation 1 **(FDR1)** $\Rightarrow$ **TFR**

see, e.g., van Zon, Cohen, PRE (2003)

Gaussian stochastic dynamics

goal: check TFR for Gaussian stochastic processes defined by the (overdamped) **generalized Langevin equation**

$$\int_0^t dt' \dot{x}(t') K(t-t') = F + \zeta(t)$$

e.g., Kubo (1965)

with Gaussian noise $\zeta(t)$ and memory kernel $K(t)$

such dynamics can generate **anomalous diffusion**:

$$\sigma_x^2 \sim t^\alpha \text{ with } \alpha \neq 1 \ (t \to \infty)$$

TFR for correlated internal Gaussian noise

consider two generic cases:

1. **internal Gaussian noise** defined by the **FDR2**,

$$< \zeta(t)\zeta(t') >\sim K(t-t') \; ,$$

with non-Markovian (correlated) noise; e.g., $K(t) \sim t^{-\beta}$

solving the corresponding generalized Langevin equation in Laplace space yields

FDR2 $\Rightarrow$ 'FDR1'

and since $\rho(W_t) \sim \varrho(x,t)$ is Gaussian

'FDR1' $\Rightarrow$ TFR

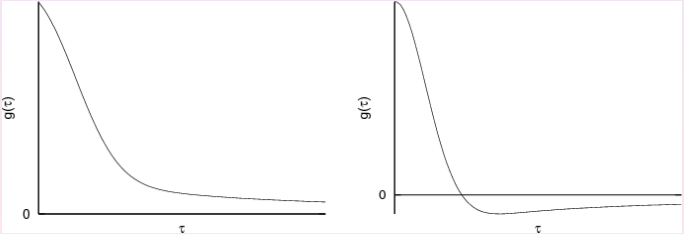
for correlated **internal Gaussian noise** $\exists$ TFR

Correlated external Gaussian noise

2. external Gaussian noise for which there is **no FDR2**, modeled by the (overdamped) generalized Langevin equation

$$\dot{x} = F + \zeta(t)$$

consider two types of Gaussian noise correlated by
$g(\tau) =< \zeta(t)\zeta(t') >_{\tau=t-t'} \sim (\Delta/\tau)^{\beta}$ for $\tau > \Delta \, , \, \beta > 0$:



persistent **antipersistent**

it is $< x >= Ft$ and $\sigma_x^2 = 2 \int_0^t d\tau (t - \tau) g(\tau)$

Results: Anomalous TFRs

σ_x^2 and the fluctuation ratio $R(W_t) = \ln \frac{\rho(W_t)}{\rho(-W_t)}$ for $t \gg \Delta$ and

$g(\tau) =< \zeta(t)\zeta(t') >_{\tau=t-t'} \sim (\Delta/\tau)^\beta$:

	persistent		antipersistent *	
β	σ_x^2	$R(W_t)$	σ_x^2	$R(W_t)$
$0 < \beta < 1$	$\sim t^{2-\beta}$	$\sim \frac{W_t}{t^{1-\beta}}$	*regime*	
$\beta = 1$	$\sim t \ln\left(\frac{t}{\Delta}\right)$	$\sim \frac{W_t}{\ln\left(\frac{t}{\Delta}\right)}$	*does not exist*	
$1 < \beta < 2$			$\sim t^{2-\beta}$	$\sim t^{\beta-1} W_t$
$\beta = 2$	$\sim 2Dt$	$\sim \frac{W_t}{D}$	$\sim \ln(t/\Delta)$	$\sim \frac{t}{\ln\left(\frac{t}{\Delta}\right)} W_t$
$2 < \beta < \infty$			$= const.$	$\sim t W_t$

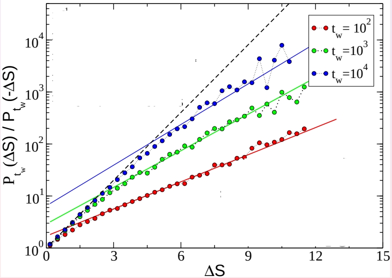
***** antipersistence for $\int_0^\infty d\tau g(\tau) > 0$ yields normal diffusion with generalized TFR; above antipersistence for $\int_0^\infty d\tau g(\tau) = 0$

Relations to experiments: glassy dynamics

$$R(W_t) = \ln \frac{\rho(W_t)}{\rho(-W_t)} = \mathbf{f}_\beta(\mathbf{t}) W_t$$

means by plotting R for different t the slope might change.

example 1: computer simulations for a binary Lennard-Jones mixture below the glass transition

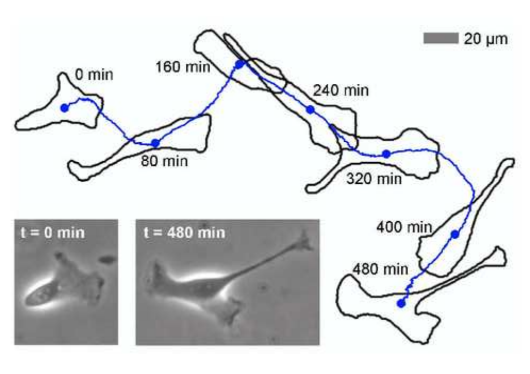


Crisanti, Ritort, PRL (2013)

- similar results for other glassy systems (Sellitto, PRE, 2009)

Biological cell migration

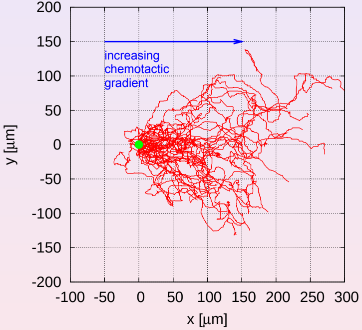
example 2: single biological cell crawling on a substrate; trajectory recorded with a video camera

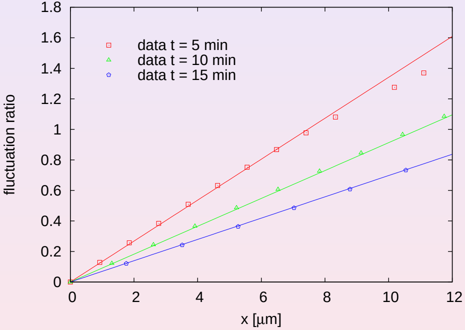


Dieterich, RK et al., PNAS, 2008

Cell migration under chemical gradients

experiments on **murine neutrophils** under **chemotaxis**:

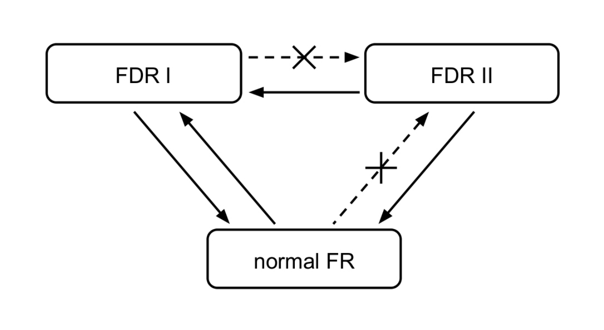




Dieterich et al. (2013)

Summary

- **relation between TFR and FDR I,II** for correlated Gaussian stochastic dynamics: (‘normal FR’= conventional TFR)



FDR2 $\Rightarrow$ FDR1 $\Rightarrow$ TFR | **$\nexists$ TFR $\Rightarrow$ $\nexists$ FDR2**

- **anomalous TFRs** likely to be important for glassy dynamics

References

- A.V. Chechkin, F.Lenz, RK, J. Stat. Mech. L11001 (2012)
- RK, A.V. Chechkin, P.Dieterich, *Anomalous fluctuation relations* in: