Constructing a Stochastic Model of Bumblebee Flights from Experimental Data

Friedrich Lenz[1] Aleksei V. Chechkin[2] Rainer Klages[1]

[1]Queen Mary University of London, School of Mathematical Sciences

[2]Institute for Theoretical Physics NSC KIPT, Kharkov, Ukraine

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Queen Mary University of London

Motivation

bumblebee foraging:

find food (nectar, pollen) in complex landscapes

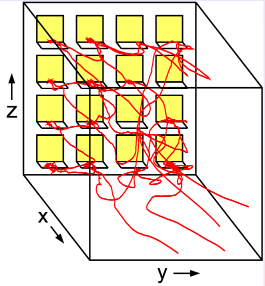
What type of motion?

Study bumblebee foraging in a *laboratory experiment*.

Bumblebee experiment

Ings, Chittka, Current Biology **18**, 1520 (2008):

bumblebee foraging in a cube of $\simeq$ 75cm side length

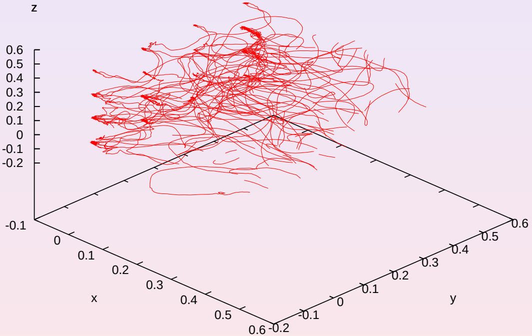


- artificial yellow flowers: 4x4 grid on one wall
- two cameras track the position (50fps) of a single bumblebee *(Bombus terrestris)*
- #bumblebees=30, #data points $\approx$ 49000

nb: Here we only focus on one aspect of the full experiment.

The main question

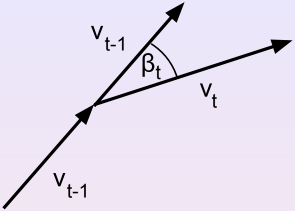
What **type of motion** do the bumblebees perform *away from the flower carpet* in terms of **stochastic dynamics**?



nb: The foraging dynamics *in interaction with the flowers* has been studied in Lenz et al., PRL **108**, 098103 (2012).

Reorientation (or CRW) model

describe biological movements in a plane by speed $s(t) = |v(t)|$ and turning angle β in comoving frame:



Correlated Random Walk model

$$\beta(t) = \xi(t) \, , \; s(t) = const.$$

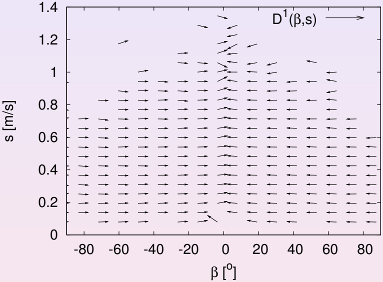
where $\xi(t)$ is typically drawn i.i.d. from a wrapped normal distribution; model captures directional biological persistence

goal: construct a **generalized CRW model from experimental data** for reproducing 'free' bumblebee flights by using Langevin-type dynamics: drift terms plus noise

$$\frac{d\beta(t)}{dt} = h(\beta(t), s(t)) + \tilde{\xi}(t)$$
$$\frac{ds(t)}{dt} = g(\beta(t), s(t)) + \psi(t)$$

Drift coefficients: phase space dynamics

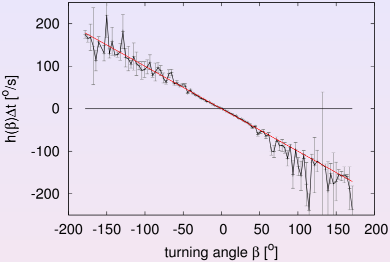
assume Markovianity for estimating **Fokker-Planck drift coefficients** h and g; normalized drift vector field:



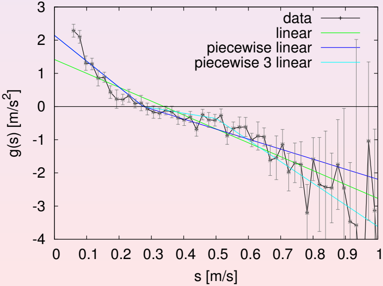
indicates that the cross-dependencies of $h(\beta(t), s(t))$ on s and of $g(\beta(t), s(t))$ on β are weak; vector field splits into

$$\begin{aligned} d\beta/dt &= h(\beta(t)) + \tilde{\xi}(t) \\ ds/dt &= g(s(t)) + \psi(t) \end{aligned}$$

Estimation of drift terms from data



extract projection $h(\beta)$ from data:
$h(\beta) \simeq -k\beta$ with $k \approx 1/\Delta t$
integrating $d\beta/dt = h(\beta(t)) + \tilde{\xi}(t)$
wrt Δt yields $\beta(t) = \xi(t)$



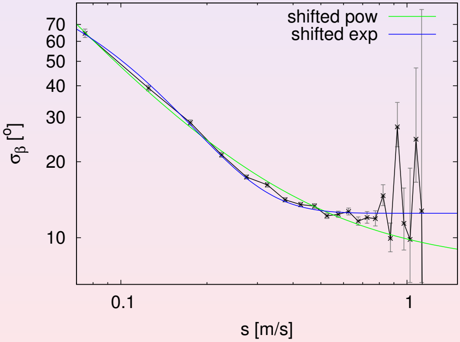
extract projection $g(s)$ from data:
$\exists$ preferred speed s_0; piecewise linear approximation for $g(s)$ in $ds/dt = g(s(t)) + \psi(t)$ yields

$$g(s) \approx (s - s_0) \cdot \begin{cases} -d_1, s < s_0 \\ -d_2, s \geq s_0 \end{cases}$$

with $d_1 > d_2 > 0$

Velocity-dependent angle noise

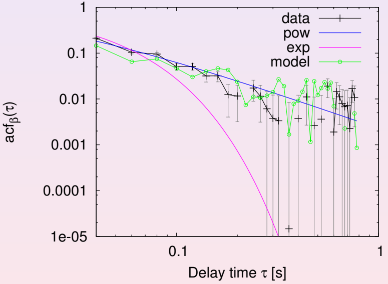
pdf for the **turning angles** β at each speed s is approximated by a Gaussian;
however, the **variance** σ_β is s-dependent (cf. naive reasoning):



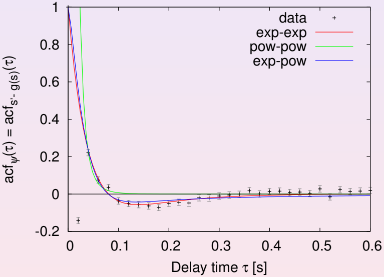
$$\begin{aligned} \beta(t) &= \xi_s(t) \\ \xi_s(t) &\sim \mathcal{N}(0, f(s(t))) \\ f(s) &= c_1 e^{-c_2 s} + c_3 \end{aligned}$$

Noise autocorrelation functions

noise $\xi_s(t)$ of turning angles β is a steep power law:



noise of speed changes $\psi(t) = ds/dt - g(s(t))$ is Gaussian with anti-correlations:



best approximated by

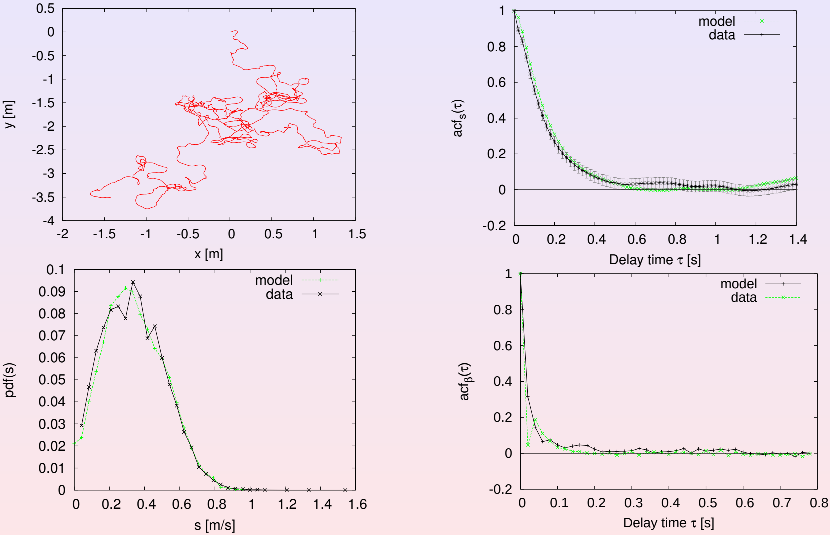
$$\mathrm{acf}_\psi(t) \approx a e^{-\lambda_1 t} + (1-a) e^{-\lambda_2 t}$$

Summary: The complete model

$$\beta(t) = \xi_s(t)$$
$$\frac{ds}{dt} = g(s(t)) + \psi(t)$$

- **turning angles** β given by power law-correlated Gaussian noise $\xi_s(t) \sim \mathcal{N}(0, \sigma_\xi(s)))$ with $\sigma_\xi(s) = c_1 e^{-c_2 s} + c_3$
- piecewise linear drift $g(s)$ for **speed** s
- ψ Gaussian noise and anti-correlated via sum of exponentials

Simulation and comparison to real data



very good agreement given the number of approximations

Summary and Reference

- We have constructed a generalized **Langevin-type correlated random walk model** that well reproduces bumblebee flights in a small cube.
- **Question 1:** Does it also work for **bee flights in the wild**?
- **Question 2:** Can we model **other animal flights** with it?

Reference:

F.Lenz, A.V.Chechkin, R.K., PLoS ONE 8, e59036 (2013)